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# A meta-heuristic algorithm for solving the road network design problem in regional contexts

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**Abstract**

In this paper we focus on the road network design problem in regional contexts. In this case a planner may have financial resources to invest for improving performance on existing roads. Against the background of an extensive literature of optimisation models and algorithms to solve this problem, the innovative aspects of this paper are as follows: the optimisation model refers to the daily operation of the network; the objective function also considers the environmental costs; the proposed meta-heuristic solution algorithm has never been used to solve this problem; and the assignment algorithm is based on Ant Colony Optimisation in order to reduce computing times. The model and algorithm were tested on a real-scale problem, showing their applicability to real dimension networks.

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*Keywords*: Transportation; Network design; Road investments; Scatter search; Ant Colony Optimisation

## 1. Introduction

Public funds available for improving rural roads are generally scarce and infrastructural investments have to be carefully planned. The aim of such investments should be to reduce not only car user costs but also social costs as whole. The latter comprise external costs, including environmental costs.

A major problem to solve is optimisation of the available resources that may be invested in the road network to minimise both private and external costs. Resources can be allocated to improving existing roads and/or to building new roads. Thus, three network design problems (NDPs) can be identified: (a) Road Improving Network

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Design Problem (RINDP), (b) Road Building Network Design Problem (RBNDP) and (c) Road Improving and Building Network Design Problem (RIBNDP). In this paper we focus on problem (a) that acts on the performances of existing roads (generally capacity and free flow speed) and we formulate it with discrete variables (more precisely with binary variables).

These problems belong to the large class of Transportation Network Design Problems (Magnanti and Wong, 1984) or to the more general Supply Design Problems (Cascetta, 2009), where road improvements and/or constructions assume the role of decision variables. Literature reviews can be found in Magnanti and Wong (1984), Yang and Bell (1998) and Feremans et al. (2003).

The general Road Network Design Problem (RNDP) has been widely studied, attracting considerable attention in the literature. Below we classify some significant papers by variables.

Discrete variable models were formulated in papers by Billheimer and Gray (1973), Boyce and Janson (1980), Poorzahedy and Turnquist (1982), Solanki et al. (1998) and Gao et al. (2005), who proposed heuristic solution algorithms, and in papers by Le Blanc (1975), Foulds (1981), Los and Lardinois (1982), Chen and Alfa (1991) and Cruz et. al. (1999), who proposed branch-and-bound solution algorithms. By contrast, meta-heuristic algorithms were proposed by Drezner and Wesolowsky (2003), Poorzahedy and Abulghasemi (2005), Ukkusuri et al. (2007) and Poorzahedy and Rouhani (2007).

Continuous variable models were formulated by Dantzig et al. (1979), Marcotte (1983), Le Blanc and Boyce (1986), Suwansirikul et al. (1987) and Meng et al. (2001), who proposed heuristic solution algorithms; Abdulaal and Le Blanc (1979), Davis (1994), Cho and Lo (1999), Chiou (2005) and Cascetta et al. (2006) proposed, instead, descent algorithms; simulated annealing approaches were suggested by Friesz et al. (1992) and Meng and Yang (2002).

Mixed variable models were formulated by Cantarella et al. (2006), Cantarella and Vitetta (2006), Gallo et al. (2010) for solving problems in urban areas with meta-heuristic approaches.

Some innovative points of this paper concerning the consolidated literature are the following: the optimisation model refers to the daily operation of the network; the objective function also considers the environmental costs; the proposed meta-heuristic solution algorithm has never been used to solve this problem; and the assignment algorithm is based on Ant Colony Optimisation (Dorigo, 1992; Dorigo and Stützle, 2004) in order to reduce computing times. The final aim is to obtain a procedure for solving the RINDP that can reach a good, even if sub-optimal, solution in acceptable computing times also for real-scale networks and for real formulations of the problem.

This paper is structured as follows: Section 2 focuses on the model formulation; the solution algorithm is described in Section 3; numerical results on a real-scale network are reported in Section 4, and Section 5 draws the main conclusions.

## 2. Optimisation model

A Road Network Design Problem (RNDP) can be generally formulated by the following constrained optimisation model:

$$\boldsymbol{y}^\wedge = \mathrm{Arg}_{\boldsymbol{y}} \min w(\boldsymbol{y}; \boldsymbol{f}^*)$$

subject to:

$$\boldsymbol{y} \in Y \quad (1)$$
$$\boldsymbol{f}^* = \boldsymbol{\Lambda}(\boldsymbol{y}, \boldsymbol{f}^*) \quad (2)$$

where: $\boldsymbol{y}$ is the vector of decision variables; $\boldsymbol{y}^\wedge$ is the optimal solution for $\boldsymbol{y}$; $w(.)$ is the objective function; $\boldsymbol{f}^*$ is the equilibrium flow vector; $\boldsymbol{\Lambda}(.)$ represents the assignment function; $Y$ is the feasible set for $\boldsymbol{y}$.

Eqn (1) summarises all constraints on decision variables. Eqn (2) represents the demand-supply consistency constraint that is in this case an equilibrium assignment constraint; this constraint links the descriptive variables, $\boldsymbol{f}^*$, to the decisional ones, $\boldsymbol{y}$, simulating user behaviour with regard to path choice on the road network.

Given a transportation supply layout (i.e. given vector $\boldsymbol{y}$), under some assumptions on cost functions and demand models, it may be proved that the equilibrium flow vector $\boldsymbol{f}^*$ exists and is unique (Cantarella, 1997; Cascetta, 2009). Therefore, eqn (2) can be considered an application: to each supply configuration, identified by vector $\boldsymbol{y}$, corresponds one and only one equilibrium link flow vector $\boldsymbol{f}^*$. The RNDP consists in searching, among all feasible supply configurations, $\boldsymbol{y}$, for the one, $\boldsymbol{y}$^, which corresponds to the optimal value of the objective function, $w(.)$.

Since our focus is on optimising scarce resources to improve road mobility at a regional level, we consider that a maximum amount of resources is available and can be used to improve the road network in a rural context. We assume that the current configuration of the road network is known and that the improvements consist in enhancing the performance of some existing roads by means of infrastructural interventions. Moreover, we assume that preliminary studies have already chosen the kind of intervention that may be provided for each road of the network, and have evaluated the corresponding costs and the corresponding benefits (in terms of free-flow speed and capacity increases).

### *2.1. Decision variables*

The decision variables identify the roads in the network to be improved: $\boldsymbol{y}$ is the vector of decision variables, $y_i$; this vector is composed by as many elements as the roads that are candidates for improvement. We assume that on each existing road, $i$, only one kind of improvement is possible; the corresponding improvement intervention is established by preliminary studies, as well as its cost ($cr_i$) and its effects on road performance. Therefore, for each road of the network the information on the possible intervention has to be known and may be arranged in a line of a table, as shown in Table 1.

Under such assumptions all variables are binary (0/1), where the value 0 indicates the current configuration for an existing road, while the value 1 indicates the corresponding improved configuration.

### *2.2. Constraints*

In this problem constraints on decision variables, a budget constraint and an assignment constraint have to be considered. The constraints on road decision variables can be written as:

$$y_i = 0/1 \qquad \forall\, i \in I$$

where $y_i$ represents the decision variable for road $i$ and $I$ is the set of roads on which it is possible to intervene. This constraint expresses the binary nature of the variables.

Table 1. Example of intervention table

| | | **Current configuration** | | **Improved configuration** | | |
|---|---|---|---|---|---|---|
| **Variable** | **Road** | *Free-flow speed, $v_0$, (km/h)* | *Road capacity, Cap, (veh/h)* | *Free-flow speed, $v_0$, (km/h)* | *Road capacity, Cap, (veh/h)* | *Costs, $cr_i$, (€)* |
| $y_1$ | 1 | 70 | 4,000 | 110 | 4,500 | 4,000,000 |
| $y_2$ | 2 | 50 | 2,000 | 70 | 4,000 | 2,800,000 |
| … | … | … | … | … | … | … |
| $y_n$ | $n$ | 40 | 1,500 | 50 | 2,000 | 1,000,000 |

The budget constraint can be written as:

$$\Sigma_i \, y_i \, cr_i \leq B$$

where $cr_i$ represents the cost of the improving intervention on road $i$. It depends on the kind of the road and on its length. The term $cr_i$ has to refer to a year, as a function of the useful life of the facility and of yearly maintenance costs. $B$ represents, similarly, the total available budget per year.

The demand-supply consistency constraint, eqn (2), links descriptive variables (traffic flows) to decision ones. In this problem we assume that demand is rigid and different for different time periods, $h$, of the year. For each time period an origin-destination vector, $\boldsymbol{d}_h$, has to be available.

Therefore, constraint (2) can be formulated as follows:

$$\boldsymbol{f}_h^* = \boldsymbol{\Lambda}(\boldsymbol{y}, \boldsymbol{f}_h^*, \boldsymbol{d}_h) \qquad \forall \, h$$

where $\boldsymbol{f}_h^*$ is the equilibrium flow vector for the time period $h$.

In this paper we adopt a Stochastic User Equilibrium (SUE) model formulated as a fixed-point problem (see Cascetta, 2009). We solve this problem with two different algorithms: the Method of Successive Averages (MSA) (Powell and Sheffi, 1982; Daganzo, 1983) and an Ant Colony Optimisation (ACO) algorithm (D'Acierno et al., 2006; 2012).

## 2.3. Objective function

The costs that we consider in the objective function are the following: private car user costs, $C_c$; resources invested, $R$; environmental costs, $EC$.

Private car user costs are the total costs incurred by car users on the road network in a year. Such costs depend on road performance, which also depends on (equilibrium) traffic flows. Since the network performances vary in the different hours of the day and in different days of the week, we have to simulate different time periods (each period is 1 hour) and sum the results for a year:

$$C_c = \Sigma_h \, (\Sigma_l \, c_l(\boldsymbol{y}, f_{l,h}^*(\boldsymbol{y})) \cdot f_{l,h}^*(\boldsymbol{y})) \cdot n_h$$

where $f_{l,h}^*$ is the flow on road link $l$ and for time period $h$, element of the equilibrium vector $\boldsymbol{f}_h^*$; $c_l(.)$ is the cost function on road link $l$; $n_h$ is the number of time periods $h$ per year.

The resources invested in the project can be expressed as (see also budget constraint):

$$R = \Sigma_i \, y_i \, cr_i$$

This term is explicitly considered in the objective function among costs, since the resources that are invested in the road network design can no longer be used for other projects that could be of interest for society.

Finally, the environmental costs are calculated as:

$$EC = \Sigma_h \, (\Sigma_l \, (Rec_{km} \cdot \Sigma_l \, f_{l,h}^*(\boldsymbol{y}) \cdot L_l) \cdot n_h$$

where, in addition to terms already defined: $Rec_{km}$ is the average environmental cost produced by a car travelling 1 km (€/km); $L_l$ is the length of road link $l$.

All terms of the objective function may be weighted so as to consider their relative importance. Hence the objective function can be summarised as follows:

$$w(\boldsymbol{y}, \boldsymbol{f}^*) = \beta_1 \cdot (\Sigma_h (\Sigma_l\, c_l(\boldsymbol{y}, f_{l,h}^*(\boldsymbol{y})) \cdot f_{l,h}^*(\boldsymbol{y})) \cdot n_h) + \beta_2 \cdot (\Sigma_i\, y_i\, cr_i) + \\ + \beta_3 \cdot (\Sigma_h (\Sigma_l (Rec_{km} \cdot \Sigma_l f_{l,h}^*(\boldsymbol{y}) \cdot L_l) \cdot n_h))$$

where $\beta_1$, $\beta_2$ and $\beta_3$ are the weights of the objective function terms. The weights should be chosen by the decision-maker according to own political objectives; for instance, a higher value of $\beta_3$ may force the results towards a solution where the environmental impacts are lower.

### 2.4. Whole model features

Some features of this model are: the decision variables are binary; the objective function is neither linear nor convex (except in particular cases); many demand assignments must be performed to evaluate each solution; the assignment constraints are not expressible in a closed form; the problem is NP-hard.

These features require efficient meta-heuristic algorithms to be defined, able to minimise solution evaluation and reduce computing times for each assignment.

## 3. Solution algorithm

The algorithm proposed for solving the RINDP is based on the meta-heuristic technique called *scatter search* (Glover et al., 2003). An application of a scatter search method to the Urban Network Design Problem was proposed by Gallo et al. (2010). The Scatter Search and its application to the proposed problem are described in the following subsections.

### 3.1. Preliminary definitions

We indicate as $\boldsymbol{y} \in Y$ a solution of a discrete optimisation problem, such as the RINDP, where $Y$ is the set of solutions. To each solution $\boldsymbol{y}$ a set of solutions $N(\boldsymbol{y}) \subset Y$ is associated, called neighbourhood of $\boldsymbol{y}$. Solution $\boldsymbol{y}$ is called the centre of the neighbourhood $N(\boldsymbol{y})$. Each solution $\boldsymbol{y}' \in N(\boldsymbol{y})$, called neighbour, is obtained from solution $\boldsymbol{y}$ by an elementary operation called "move"; a move changes only one value of a variable of solution $\boldsymbol{y}$, generating the next solution $\boldsymbol{y}'$. Usually it is assumed that the neighbourhoods are symmetrical, that is: if $\boldsymbol{y}' \in N(\boldsymbol{y})$ then $\boldsymbol{y} \in N(\boldsymbol{y}')$.

A solution $\boldsymbol{y}_{loc}\hat{} \in Y$ is a local optimum if the objective function value $w(\boldsymbol{y}_{loc}\hat{})$ is less [greater] than, or equal to, in a minimisation [maximisation] problem, objective function values corresponding to all solutions belonging to its neighbourhood:

$$w(\boldsymbol{y}_{loc}\hat{}) \leq w(\boldsymbol{y}') \quad \forall\, \boldsymbol{y}' \in N(\boldsymbol{y}) \qquad\qquad [w(\boldsymbol{y}_{loc}\hat{}) \geq w(\boldsymbol{y}') \quad \forall\, \boldsymbol{y}' \in N(\boldsymbol{y})]$$

The distance of solution $\boldsymbol{y}''$ from solution $\boldsymbol{y}'$ is the minimum number of moves needed to transform solution $\boldsymbol{y}''$ into solution $\boldsymbol{y}'$; the distance is indicated with $D(\boldsymbol{y}''-\boldsymbol{y}')$. Any solution belonging to a neighbourhood has a distance equal to 1 from the centre.

For the RINDP a solution $\boldsymbol{y}$ represents a configuration of the network. In our problem the current solution, $\boldsymbol{y}^0$, is the $\mathbf{0}$ vector. Conventionally, we assume as positive a move that converts the value of a variable $y_i$ from 0 to 1; the opposite moves are assumed negative.

### 3.2. Neighbourhood Search

If $\boldsymbol{y}^k$ is a solution, the Neighbourhood Search generates the following solution $\boldsymbol{y}^{k+1}$ such that:

$$\boldsymbol{y}^{k+1} \in N(\boldsymbol{y}^k)$$

and $\boldsymbol{y}^{k+1}$ respects a specified rule. One of the most commonly adopted rules for generating the next solution is the *steepest descent method* (SDM); it examines all neighbours, calculating their objective function values, and chooses the next solution as the one with the best value:

$$w(\boldsymbol{y}^{k+1}) = \text{Min } \{w(\boldsymbol{y}); \quad \forall\, \boldsymbol{y} \in N(\boldsymbol{y}^k)\}$$

The procedure then generates at each iteration a solution better than the previous one choosing, among all solutions belonging to the neighbourhood, the one with the best objective function value. The procedure ends when solution $\boldsymbol{y}^k$ is a local optimum.

This method is not suitable for our problem if the network has real dimensions. Indeed, in this case the variables can be very numerous and the neighbourhoods can be very wide; evaluating at each step the objective function for all neighbours is not compatible with acceptable computation times, since each objective function evaluation requires that several equilibrium assignments be solved, one for each time period. In order to reduce the computation times, it is possible to use a random method for generating the following solution, called the *random descent method* (RDM). This method randomly extracts a solution from the neighbourhood and determines its objective function; if the new solution is better than the current one and is feasible, it becomes the current solution; otherwise, another neighbourhood solution is randomly extracted and so on, until a better solution is found. If no feasible neighbours improve the objective function, the solution is a local optimum.

### *3.3. Scatter Search*

Scatter search is a *meta-heuristic* technique for solving complex combinatorial optimisation problems. It can be adapted in several ways to several kinds of optimisation problems by suitably defining the criteria used in the *phases* of the solution procedure. A phase of Scatter Search is a mathematical or algorithmic subroutine that operates on a solution subset, generating another solution subset. Below, the scatter search phases are examined and adapted to the RINDP.

*Phase 1 – Starting set generation.* In this phase a set of solutions is generated which should have a high level of *diversity* so as to cover different regions of the solution set. The subroutine that generates the starting set is also called the *Diversification Generation Method*. This routine is applied in our problem as follows:

- we define a mother solution as the initial configuration of the road network, $\boldsymbol{y}^0$;
- from this solution, other solutions, at fixed a priori distances from the mother solution, are randomly generated; they are called base solutions;
- unfeasible solutions are eliminated and substituted with other solutions (randomly generated) at the same distance from the mother one;
- the mother solution and the base solutions constitute the starting set.

*Phase 2 – Improvement in current solutions.* In this phase, from any current solution an improved solution is generated by an algorithmic subroutine that is also called the *Improvement Method*. Several improvement methods can be adopted; in this paper we will test both the *steepest descent method* (SDM) and the *random descent method* (RDM) introduced in Section 3.2. All improved solutions are local optima.

*Phase 3 – Reference set generation or updating.* A *reference set* is generated by selecting all improved solutions (local optima) generated in the previous phase or, if they are too numerous, only part of them; in this second case, the selection should take account of objective function values (*good solutions*) and diversity (*scattered solutions*). The reference set will consist of good solutions with better values of the objective function, and scattered solutions with maximum distances from the best solution, until the maximum number of solutions is reached. The subroutine that generates or updates the reference set is called the *Reference Set Update Method*. In the tests reported in Section 4 we consider all different local optima generated in the previous phase as reference set solutions.

*Phase 4 – Solution subset generation.* In this phase some solution subsets are generated, consisting of some solutions belonging to the reference set, which will be *combined* in the subsequent phase to generate other solutions. The subsets may be generated in several ways. In the tests reported in Section 4 we propose to assume the maximum number of subsets equal to 4 and to generate the subsets adding to the best solution in the reference set three other randomly extracted solutions.

*Phase 5 – Solution combination.* In this phase, the solutions of each subset are combined. The *Solution Combination Method* may differ depending on the kind of problem, and usually leads to one solution being generated from each subset. The method generally associates a *score* to each value that can be assumed by a variable $y_i$; this score has to take account of objective function values of solutions of the subset and of the times that the specific value is assumed by the variable in all solutions belonging to the subset. The combined solution obtained from the subset will be that in which every variable assumes the value with the best score. In our problem the new solution is generated as follows:

- a *solution score* is associated to each solution of the subset as follows:
  the ratio between the objective function value corresponding to the solution and the sum of objective function values of all subset solutions is calculated (objective function ratio); since we are dealing with a minimisation problem the solution score is calculated as 1 less the objective function ratio;
- a *variable value score* is associated to each value (0 or 1) that can be assumed by a variable $y_i$ as the sum of the relative values of solutions in which that variable assumes that specific value;
- the combined solution is generated such that any variable, $y_i$, assumes the best variable value score.

The solutions obtained in phase 5 are improved (phase 2), generating a new reference set. The procedure ends when the reference sets in two successive iterations are equal or when a fixed a priori number of iterations is reached. All solutions belonging to the last reference set are local optima. In the tests reported in Section 4 we stopped the algorithm at the end of phase 5 when at least 5,000 solutions had been evaluated.

## 4. Numerical results

The proposed model and algorithm were tested on a real-scale network (see Figure 1a), namely the regional road network of Campania. The network graph consists of 91 centroids, 161 connectors, 262 road nodes, 764 road links and represents over 8,700 kms of roads.

We consider 8 different time periods: work-day morning peak hour; work-day afternoon peak hour; work-day day-time off-peak hour; work-day night-time hour; pre-holiday day-time hour; pre-holiday night-time hour; holiday day-time hour; holiday night-time hour.

Therefore, travel demand among traffic zones is represented by eight hourly matrices, one for each time period, *h*; the matrices were estimated from a previous study on the mobility of the region of Campania available to the authors. The number of hours per year and total trips in the corresponding hourly origin destination matrices are reported in Table 2.

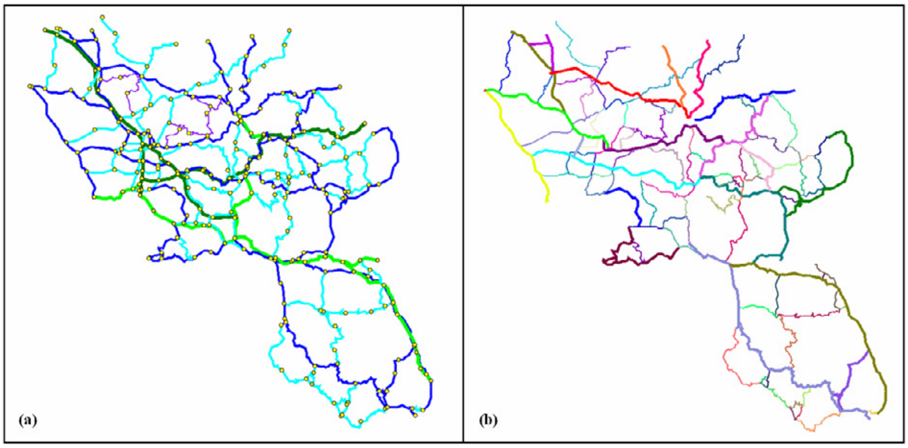


Fig. 1. (a) real-scale test network; (b) road sections as problem variables

Table 2. Main features of time periods

| Time period | Hours per year | Total trips [veh/h] |
|---|---|---|
| Work-day morning peak hour matrix | 486 | 165,328 |
| Work-day afternoon peak hour matrix | 729 | 115,728 |
| Work-day day-time off-peak hour matrix | 2,187 | 66,130 |
| Work-day night-time hour matrix | 2,430 | 24,799 |
| Pre-holiday day-time hour matrix | 720 | 82,664 |
| Pre-holiday night-time hour matrix | 720 | 33,066 |
| Holiday day-time hour matrix | 744 | 66,130 |
| Holiday night-time hour matrix | 744 | 16,533 |

The test network links are classified according to the following kinds of roads: main motorways; secondary motorways; main rural roads; secondary rural roads; local rural roads. We assume that the regional road authority may intervene, with improvements, on all existing roads except for motorways. Moreover, we define a section as a sequence of links that belong to the same rural road (on both directions) and that has to be considered jointly (an improvement is either envisaged for all section links or for no section links); therefore, we introduce a variable $y_i$ for each section. We identify 20 sections (decision variables from $y_1$ to $y_{20}$) for the main rural roads and 82 other sections (decision variables from $y_{21}$ to $y_{102}$) for the other rural roads; our problem thus presents 102 decision variables. Figure 1b reports the variables identified, where the bold lines refer to the main rural roads. We assume that weights of the objective function terms are all equal to 1.

The first numerical results concern the evaluation of benefits of the random descent (RDM) vs. steepest descent (SDM) methods, as neighbourhood search procedures, and of ACO vs. MSA, as assignment algorithms. We thus tested four local search algorithms, in the case of the work-day morning peak hour, in order to ascertain the best to adopt within the proposed scatter search procedure. The proposed local search algorithms are as follows: MSA_SDM, MSA_RDM, ACO_SDM and ACO_RDM. In Table 3 the main results for the tested algorithms are summarised. It may be noted that these algorithms generally lead to different final solutions; only MSA_SDM and ACO_SDM lead to the same solution (except for small differences due to the stop threshold) since they are based on the steepest descent method.

The shapes of objective function reductions for the four algorithms are reported in Figure 2. It can be noted that algorithms based on random descent methods allows objective function values to be reduced, examining a significantly lower number of solutions, reducing computational effort as much as ten-fold.

Table 3. Performance of local search algorithms

| Algorithm | O.F. Value [M€/year] | Number of network loadings | Number of examined solutions | Computing times [h] | Number of improved roads |
|---|---|---|---|---|---|
| MSA_SDM | 39,881 | 62,676 | 5,918 | 11.61 | 53 |
| MSA_RDM | 39,855 | 8,525 | 788 | 1.58 | 55 |
| ACO_SDM | 39,798 | 50,129 | 5,918 | 9.23 | 53 |
| ACO_RDM | 39,712 | 6,018 | 660 | 1.10 | 56 |

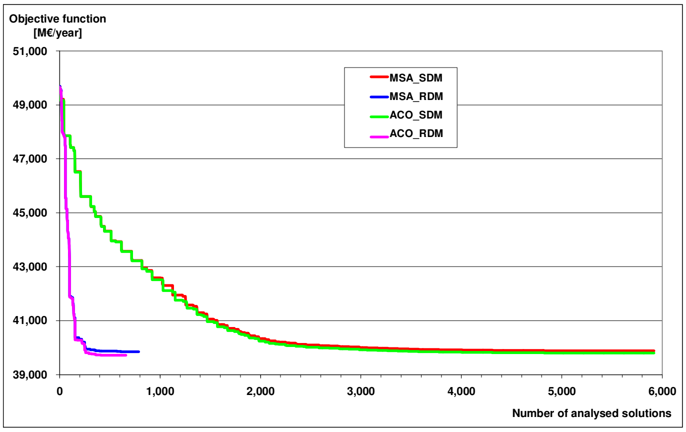


Fig. 2. Comparison of proposed local search algorithms in terms of objective function reduction

Moreover, we compared results between the application of ACO_RDM in the case of the work-day morning peak hour matrix and in the case of whole-week matrices. Obviously, all results are scaled to the same temporal level (i.e. a year). The results are summarised in Table 4. Significant differences can be identified: the number of improved roads decreases for whole-week demand vs. peak-hour. It shows that considering only peak-hour periods for network design may lead to overestimating the advantages of investments in the road network.

After those preliminary tests, we applied the scatter search technique, adopting a value for the budget constraint equal to 100 M€/year. In all, scatter search examined 6,136 solutions in about 42 hours. An exhaustive approach is not possible since the solutions to examine are $2^{102}$ and the required computing time will be about $4.18 \cdot 10^{18}$ million years. The best local optimum corresponds to an objective function value of 12,807.35 M€/year, with a reduction against the starting solution (no-intervention solution) of about 6.37 %. Comparison between single terms of the objective function is reported in Table 5 and shows a reduction in private car user costs, $C_c$, of about 9.71 % and a reduction in environmental costs, $EC$, of about 3.39 %. Moreover, it can be noted that the budget constraint is not active, since the resources invested, $R$, are equal to 88.41 M€/year vs. a constraint value of 100 M€/year.

Table 4. Peak hour vs. whole-week transportation demand: results adopting ACO_RDM

| Transportation demand | O.F. Value [M€/year] | Number of network loadings | Number of examined solutions | Computing times [h] | Number of improved roads |
|---|---|---|---|---|---|
| Peak hour | 39,704 | 4,835 | 660 | 0.89 | 56 |
| Whole-week | 12,764 | 15,288 | 481 | 3.05 | 31 |

Table 5. Comparison of starting and best solutions

| Solution | Private car user costs [M€/year] | Environmental costs [M€/year] | Resources invested [M€/year] | Objective function value [M€/year] |
|---|---|---|---|---|
| Starting solution | 7,830.09 | 5,848.41 | - | 13,678.51 |
| Best local optimum | 7,068.96 | 5,649.98 | 88.41 | 12,807.35 |
| *Percentage variation* | *- 9.72 %* | *- 3.39 %* | - | *- 6.37 %* |

The objective function reduction is reported in Figure 3a. Scatter search generated 14 different local optima. Interestingly, all 14 local optima correspond to different solutions of the problem: some of them correspond to similar values of objective functions but others correspond to solutions with objective function values which worsen until 3.2 % with respect to the best one.

The final solution provides for the improvement of 34 roads, highlighted in Figure 3b. Examination of the best solution shows that many improved roads are located in the more congested areas of the Campania, chiefly in the province of Naples.

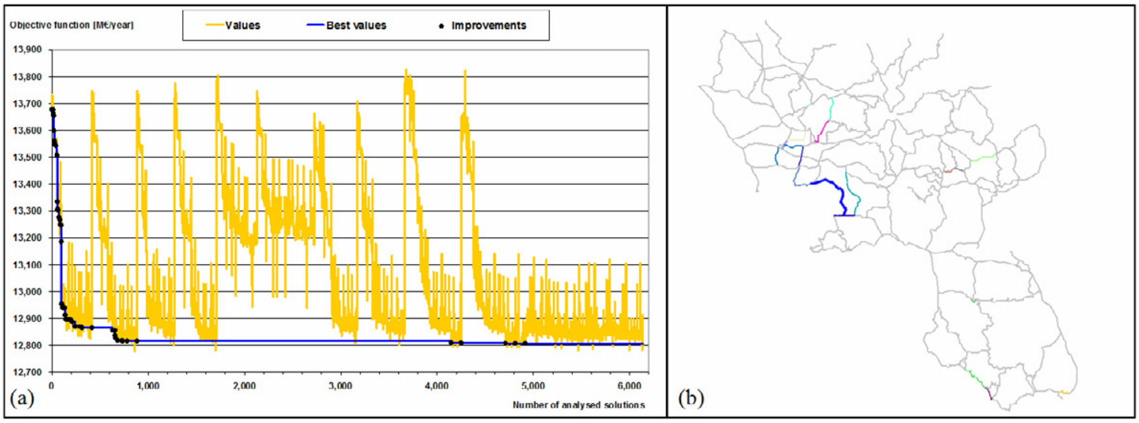


Fig. 3. (a) objective function reduction; (b) roads improved in the best solution

## 5. Conclusions

This paper proposes a model and a metaheuristic algorithm for solving the road network design problem in a regional context, where a planner has to evaluate the optimal allocation of scarce resources in the improvement of existing roads. The model is able to simulate different time periods so as to consider properly the impact of congestion on the design. Indeed, designing a road network only for the peak-hour may overestimate the advantages of infrastructural investments.

In order to solve the problem a Scatter Search algorithm was proposed and tested on a real-scale network. To reduce computing times a random descent method for improving solutions and an ACO-based assignment algorithm were proposed as subroutines of the Scatter Search. Numerical results showed the applicability of the procedure on real-scale networks in computing times compatible with long-term planning purposes.

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